

An introduction to elliptic filters for the radio amateur

by JOHN WILKINSON, BSc, G4HGT*

Introduction

Elliptic function filters possess the best attenuation vs frequency characteristics of all passive filters, and can give superb performance when a sharp cut-off is required. Unfortunately, little information on their design exists in amateur literature, and it is often difficult for the non-engineering amateur to sort out the mass of information presented in filter handbooks.

This article is split into two parts:

- (a) A brief description of the filters and practical designs with scaling methods for low- and high-pass configurations.
- (b) A more thorough treatment of filter tables and methods for bandpass filters.

In this way it is hoped that these useful filters can be tried with some degree of confidence.

Basic principles

An elliptic filter differs from other types, such as Butterworth and Chebyshev, in that it has resonant circuits in it. These provide points in the amplitude vs frequency response where the signal attenuation is very high. Due to these transmission zeros, the steepest rate of attenuation vs frequency can be achieved.

Fig 1 compares the three common filter configurations. The elliptic filter is approximately 15dB better than the Chebyshev filter at 1.5 times the turnover frequency ω_0 , but above this frequency the attenuation does not continue to fall away. This is the price that has to be paid for having a sharp turnover, although normally the filter designer can take this into account when a specification is to be met. Careful design can place these zeros to give high attenuation at desired points; in the examples, one design is given having zeros at $2\omega_0$ and $3\omega_0$, an ideal response for most harmonic-producing transmitters.

Like Chebyshev filters, elliptic filters do not have a flat response in the passband. Instead, they are equi-ripple, with the amount of ripple being chosen by the designer. Values of ripple less than 1dB are typical, with increased passband ripple being traded against increased stopband attenuation.

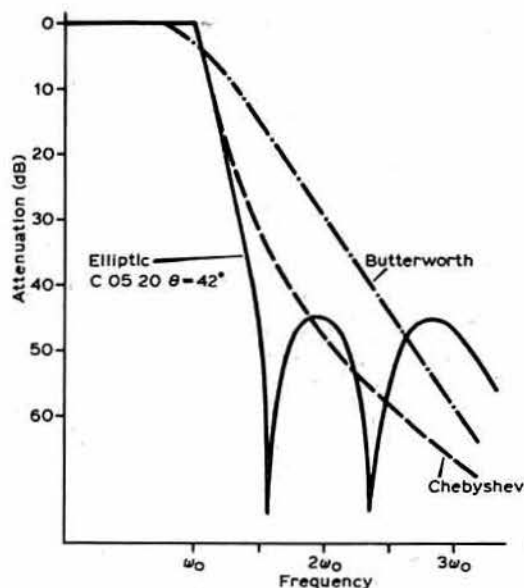


Fig 1. Comparison of filter types

Fig 3. (a) Frequency response of C 05 05 $\theta = 31^\circ$
(b) Component values for C 05 05 $\theta = 31^\circ$

*10 Ringway Road, Lee Park, Liverpool, Merseyside L25 3QS.

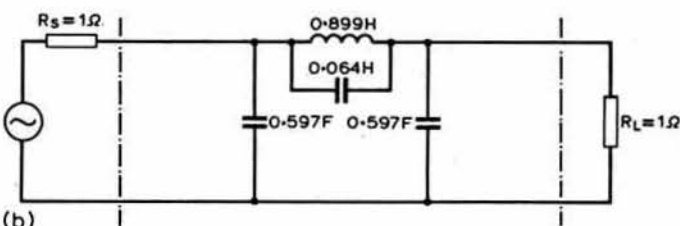
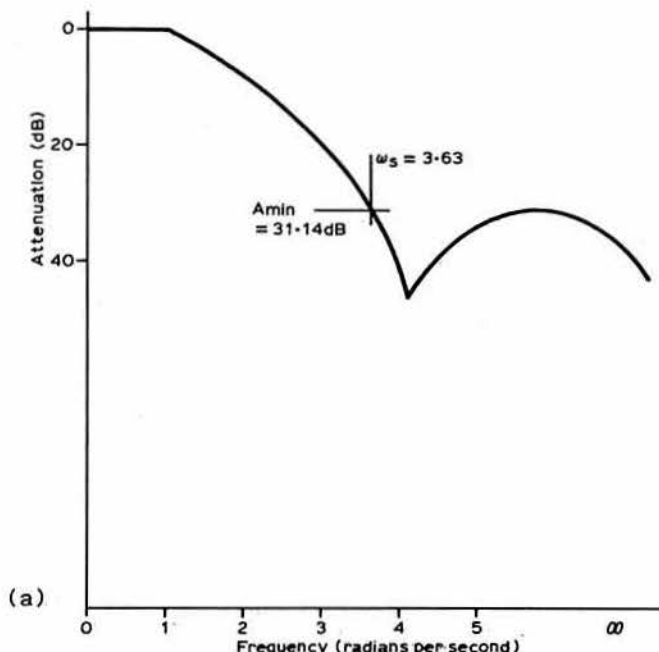
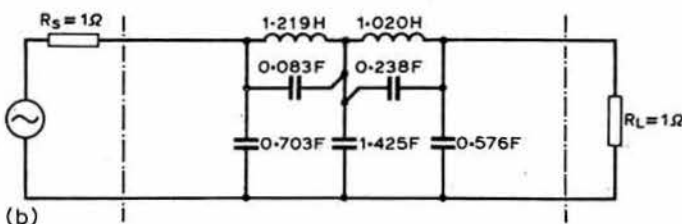
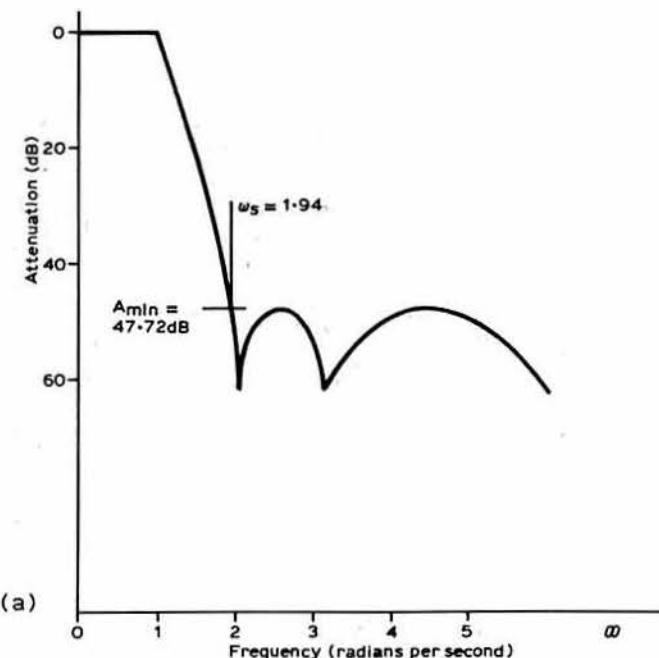


Fig 2. (a) Frequency response of C 03 05 $\theta = 16^\circ$
(b) Component values for C 03 05 $\theta = 16^\circ$ (Capacitor value is 0.064F, not 0.064H)



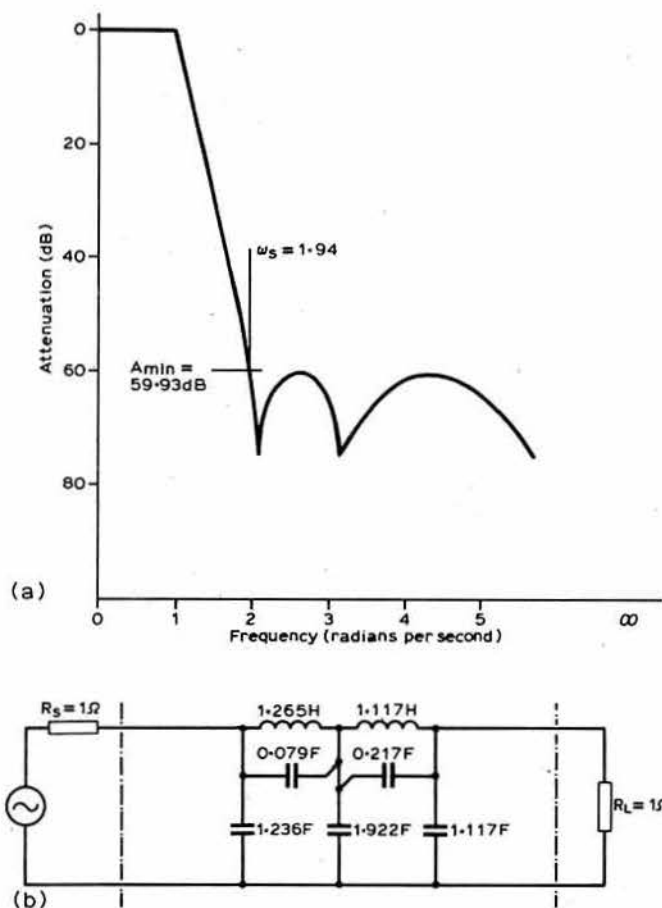


Fig 4. (a) Frequency response of C 05 20 $\theta = 31^\circ$
(b) Component values for C 05 20 $\theta = 31^\circ$

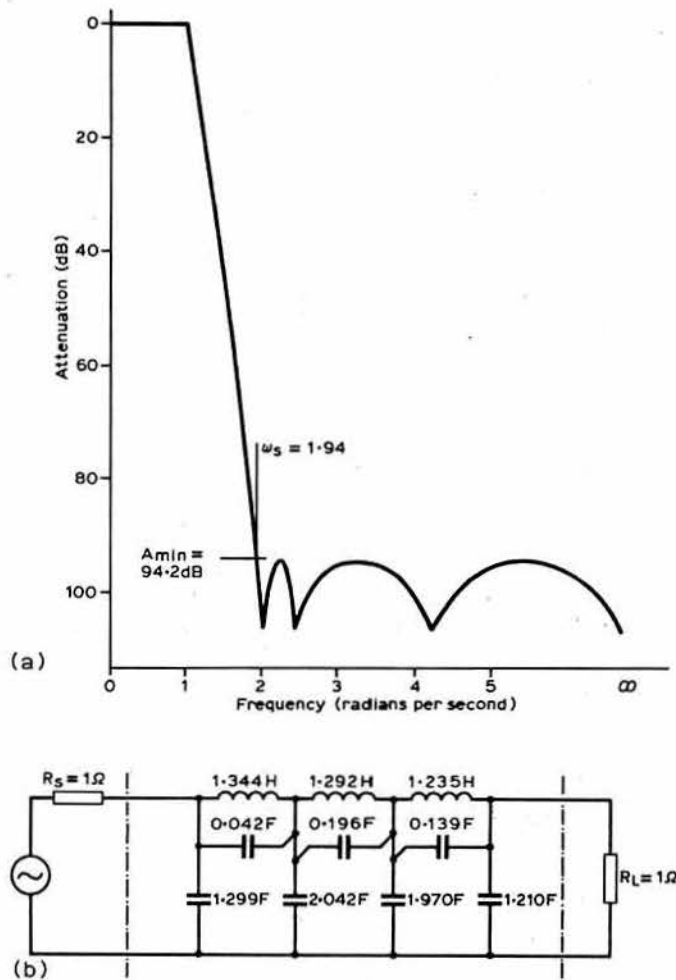


Fig 5. (a) Frequency response of C 07 20 $\theta = 31^\circ$
(b) Component values for C 07 20 $\theta = 31^\circ$

Normalization

Filter tables are printed in handbooks on filters (see "Bibliography") and detail the component values and frequency responses of a selection of filters. They are arranged so that all the filters have a turnover frequency of 1 rad/s, usually denoted ω_0 . (Angular frequency is a scaled version of frequency more usually expressed in hertz: 1 rad/s = 2π cycles/s = 6.28 Hz.) The frequency response is then expressed in terms of ω_0 , ie 40 dB of attenuation at $3.75\omega_0$ etc. This technique, known as normalization, reduces the number of filters that have to be tabulated.

The values of inductors and capacitors obtained from tables are measured in Henries and Farads respectively, and the normalized system impedance is 1Ω.

Frequency and impedance scaling

Once a normalized frequency response has been obtained, either from tables or the examples, the values of inductors and capacitors have to be adjusted to give the desired response at the frequency and impedance level required.

For frequency scaling, the following rules are applied:

$$C(\text{scaled}) = \frac{C(\text{table})}{2\pi \times \text{desired frequency}}$$

and

$$L(\text{scaled}) = \frac{L(\text{table})}{2\pi \times \text{desired frequency}}$$

(If a 2.4732F capacitor is required in the normalized table, then for a 28MHz 1Ω filter its value will be $\frac{2.4732}{2\pi \times 28 \times 10^6} = 14.05\text{nF}$.)

The filter can then be impedance scaled as follows:

$$C(\text{used}) = \frac{C(\text{scaled})}{\text{impedance}}$$

and

$$L(\text{used}) = L(\text{scaled}) \times \text{impedance.}$$

(So in a 50Ω system, the above capacitor would have the value:

$$C = \frac{14.05\text{nF}}{50} = 281\text{pF}.)$$

Note that the resonant frequencies of the tuned circuits so formed should still be in the same ratios to the turnover frequency as they were in the normalized circuit. Due to the transformations used, the following equation should be used:

$$\omega = \frac{1}{\sqrt{L(\text{table}) \times C(\text{table})}}$$

Definition of terms

In the examples, each filter is denoted by a code of the form:

$$C \ n \ \rho \ \theta = m$$

and these completely describe the filter.

n is the order of the filter and gives an indication of the quality and hence complexity of the filter.

ρ is the reflection coefficient of the filter and is related to the passband ripple and the vswr.

θ determines the steepness of the filter—the higher θ , the steeper the filter.

As well as the values of inductors and capacitors required, a filter table will also give values for tuned circuit resonant frequencies, and an indication of the stopband attenuation.

Unlike Butterworth and Chebyshev filters, the turnover frequency ω_0 is measured at the attenuation = passband ripple point, as opposed to the 3dB attenuation point. The frequency at which the attenuation first equals the stopband attenuation is denoted ω_s , and $A_{\min}$ is the minimum attenuation in the stopband.

How to use the examples

The purpose of this section is to simplify the work needed to produce an elliptic filter. Figs 2-5 are standard filter designs taken from filter tables, and they should satisfy filter requirements for most amateurs. The filters are presented in normalized form, ie turnover frequency = 1 rad/s, and a graph of expected attenuation vs frequency is also given. Methods for

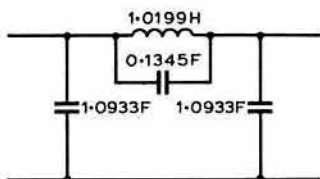


Fig 6. Normalized component values for C 03 20 $\theta = 25^\circ$

converting these designs to practical filters have been explained in earlier sections, but to give the reader more confidence a full worked example of a third-order design will be given.

In the examples, it will be noted that even-order filters are omitted. This is because they have the same number of inductors as the filter with the next higher order, but due to their lower order their performance is not as good. All the scaling methods outlined will work with even-order filters, and the interested amateur will find them useful when the filter is to be used between non-equal terminations.

Worked example

A filter is required for the output of a 10MHz transmitter. A turnover frequency of 11MHz is desirable and more than 30dB of attenuation is required at the third harmonic. The system impedance is 50Ω.

This specification is met by the filter denoted C 03 20 $\theta = 25^\circ$; its circuit is shown in Fig 6.

For an 11MHz filter:

$$C_1 = C_3 = \frac{1.0933}{2 \times \pi \times 11 \times 10^6} = 15.82 \text{ nF}$$

$$C_2 = \frac{0.1345}{2 \times \pi \times 11 \times 10^6} = 1.946 \text{ nF}$$

$$L = \frac{1.0199}{2 \times \pi \times 11 \times 10^6} = 14.75 \text{ nH}$$

In a 50Ω system:

$$C_1 = C_3 = \frac{15.82 \text{ nF}}{50} = 316.4 \text{ pF}$$

$$C_2 = \frac{1.946 \text{ nF}}{50} = 38.92 \text{ pF}$$

$$L = 14.75 \text{ nH} \times 50 = 737 \text{ nH}$$

The resonant frequency of the tuned circuit is:

$$f_s = \frac{1}{2\pi\sqrt{LC_2}} = 29.706 \text{ MHz}$$

Above this frequency the attenuation is greater than 31.4dB.

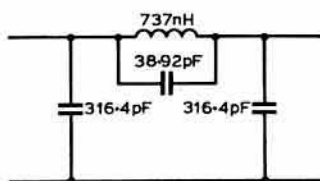


Fig 7. Final values for the worked example

Duality

In the above examples, the circuit used has the minimum number of inductors. The dual circuit has the maximum number of inductors and is produced by the following rules:

- shunt circuit becomes series circuit;
- parallel tuned circuit becomes series tuned circuit;
- capacitors become inductors and vice versa;
- all values remain the same.

Fig 8 shows the method for a fifth-order filter.

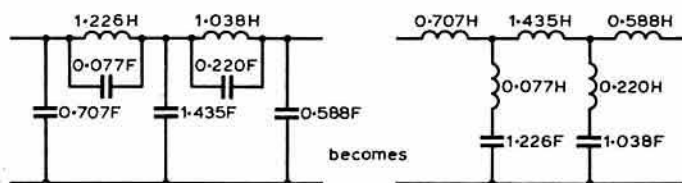


Fig 8. Duality transformation

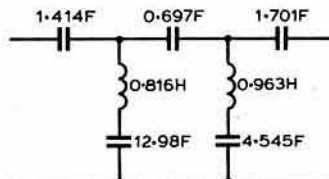


Fig 9. Highpass transformation of Fig 8

Highpass transformation

This is best achieved by using the dual circuit as shown above. The transformation rules are:

L becomes C and C becomes L

and the values are inverted, ie $L = \frac{1}{C}$

The highpass filter produced has the minimum number of inductors and the resulting filter is shown in Fig 9.

How to find your own filter design

There are quite a few handbooks (see bibliography for details) containing tables of values for elliptic filters, and if the reader is interested in designing his own particular filter then this section may clarify the methods to be used. As mentioned earlier, three parameters are used by mathematicians to denote a particular filter, these are; n , p and θ , being order, reflection coefficient and modular angle respectively. Each table is headed C n p , and the filters are tabulated in terms of θ . (C denotes Caer who was a famous network theorist).

The order n of a filter indicates the number of transmission zeros as follows:

$$\text{number of zeros} = \frac{n+1}{2} \text{ for } n \text{ odd}$$

$$= \frac{n}{2} \text{ for } n \text{ even.}$$

The number of tuned circuits is always one less than the number of transmission zeros, due to the fact that there is always a transmission zero at infinite frequency (assuming low-pass basis):

$$\text{number of tuned circuits} = \frac{n-1}{2} \text{ for } n \text{ odd,}$$

$$= \frac{n-2}{2} \text{ for } n \text{ even.}$$

A disadvantage of even-order filters is their need for an extra inductor; it is therefore sensible to use the next higher odd-order filter and benefit from its improved performance. For filters requiring non-equal terminations, and if a small passband loss can be tolerated, even-order filters come into their own.

Reflection coefficient p will probably be understood more easily as a form of vswr:

$$p(\%) = \frac{\text{VSWR} - 1}{\text{VSWR} + 1} \times 100\%;$$

and p is directly related to the passband ripple R_{dB} by:

$$R_{dB} = -10 \log(1 - p^2).$$

Filter tables usually take values of p in steps of five per cent for most usable values.

Table 1. Values of VSWR and R_{dB} in terms of p

$p(\%)$	VSWR	R_{dB}
1	1.020	0.0004
5	1.105	0.011
10	1.222	0.044
15	1.353	0.099
20	1.500	0.177
50	3.000	1.250

As was mentioned before, p also affects the stopband attenuation, with increased attenuation being gained at the expense of matching.

The modular angle θ of the filter determines the steepness of the filter and is defined as:

$$\theta = \arcsin \frac{1}{\omega_s}$$

and tabulation is in steps of 1° ; see Table 2.

Table 2. Values of ω_s in terms of θ

θ	ω_s
10	5.76
30	2.00
50	1.31
70	1.06
90	1.00

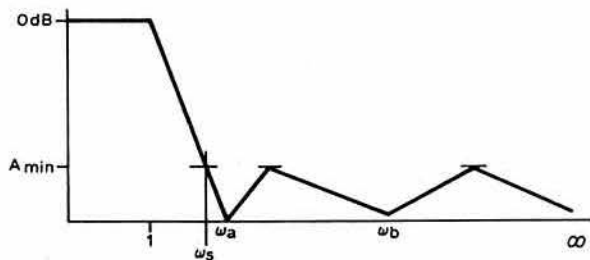


Fig 10. Showing relevant frequency response features

Again, there is a compromise to be made between stopband attenuation and steepness, since for high values of θ the stopband performance is degraded. Thus if a very sharp filter is required, then increasing the order is the only way to obtain a high stopband attenuation.

A typical normalized filter design table will have the form shown in Table 3.

Bandpass filters

Elliptic function lowpass filters can be transformed into bandpass filters which are superior to other types such as Butterworth or Chebyshev. Each lowpass transmission zero is transformed into a pair of transmission zeros positioned either side of the centre frequency. Bandpass filters are geometrically symmetrical about their centre frequency; that is, two frequencies having the same level of attenuation on either side of the passband are related by the equation:

$$f_{\text{upper}} \times f_{\text{lower}} = f_0^2$$

If only one stopband frequency is known then the other can be found.

The lowpass filter is transformed by resonating each inductor with a series capacitor, and each capacitor with a parallel inductor at the centre frequency f_0 ; the transformations are shown in Fig 11.

The last network in the figure occurs frequently in practice, and the configuration is not very easy to implement. There are two parallel resonances which are not independent. It is possible to use this network, but an alternative configuration exists which uses two parallel tuned circuits in series. These correspond to the two stopband zeros, one above and one below the passband.

The transformation is achieved as follows:

First calculate,

$$\beta = \left[1 + \frac{1}{2LC} \right] + \sqrt{\left[1 + \frac{1}{2LC} \right]^2 - 1}$$

Then,

$$L_a = \frac{1}{C(\beta + 1)} \quad L_b = \beta L_a$$

$$C_a = 1/L_b \quad C_b = 1/L_a$$

the resonant frequencies are;

$$\Omega_a = \sqrt{\beta} \quad \text{and} \quad \Omega_b = 1/\Omega_a$$

Example of low to bandpass transformation

A filter is required for the input of a 3.5MHz receiver. The desired frequency response is shown in Fig 13, and 30dB of attenuation is required at 3.8725MHz.

First calculate

$$f_0 = \sqrt{3.500 \times 3.800} = 3.6469 \text{ MHz}$$

The lower stopband frequency then follows at 3.4345MHz.

The filter steepness is calculated from;

$$\text{Steepness} = \frac{\text{stopband bandwidth}}{\text{passband bandwidth}} = \frac{3.8725 - 3.4345}{0.3000} = 1.4601$$

A lowpass filter is chosen having a steepness of 1.4601 and possessing the desired attenuation. From Table 3, fifth order filter C 05 050 = 43° has been selected.

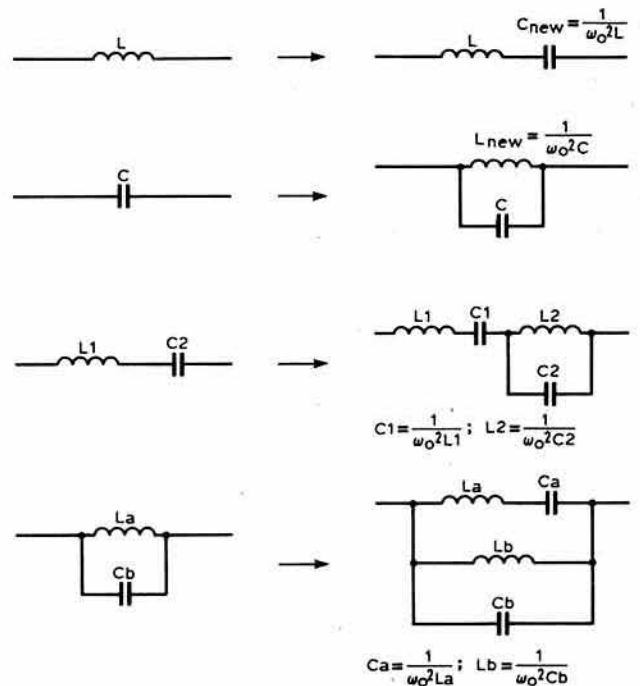
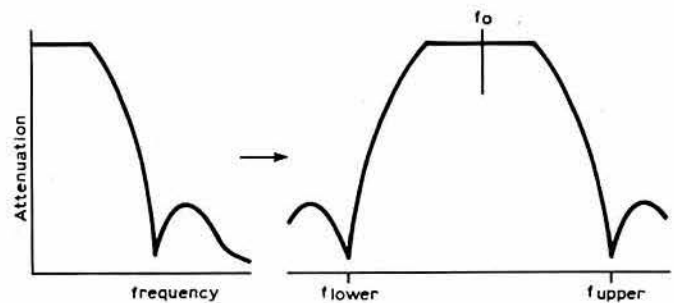


Fig 11. Low to bandpass transformations

The network values are now multiplied by Q_{bp} . This is calculated from:

$$Q_{bp} = \frac{f_0}{\text{bandwidth}} = \frac{3.6469}{0.3} = 12.1564$$

Now form a normalized bandpass filter using the transformations shown in Fig 11, assuming $\omega_0 = 1$ throughout.

The upper branches are now transformed as follows: For the left-hand branch,

$$L = 13.7270H$$

$$C = 2.0933F$$

$$\beta = \left[1 + \frac{1}{2LC} \right] + \sqrt{\left[1 + \frac{1}{2LC} \right]^2 - 1}$$

$$\beta = 1.20476$$

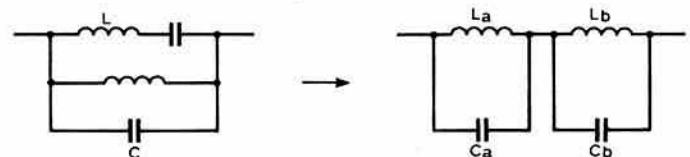


Fig 12. Transformation of upper branches

Table 3. Typical normalized filter design parameters

θ	ω_s	$A_{\min}$	ω_a	ω_b	C1	C2	C3	C4	C5	L2	L4
40.0	1.5557	35.85	1.6170	2.4377	0.6580	0.1456	1.3282	0.4590	0.4426	1.1556	0.8331
41.0	1.5243	34.67	1.5833	2.3781	0.6521	0.1541	1.3168	0.4924	0.4252	1.1472	0.8099
42.0	1.4945	33.52	1.5515	2.3213	0.6460	0.1630	1.3053	0.5283	0.4072	1.1384	0.7862
43.0	1.4663	32.38	1.5213	2.2672	0.6397	0.1722	1.2937	0.5669	0.3885	1.1292	0.7620
44.0	1.4396	31.27	1.4926	2.2154	0.6332	0.1819	1.2822	0.6085	0.3693	1.1198	0.7375
45.0	1.4142	30.17	1.4654	2.1660	0.6265	0.1920	1.2706	0.6535	0.3494	1.1099	0.7125

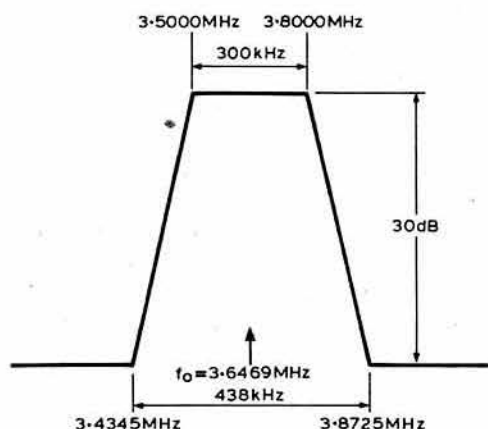


Fig 13. Filter frequency response

$$L_a = \frac{1}{C(\beta + 1)} = 0.21667H$$

$$L_b = \beta L_a = 0.26104H$$

$$C_a = \frac{1}{L_b} = 3.83088F$$

$$C_b = \frac{1}{L_a} = 4.61529F$$

$$\Omega_a = 1.09762,$$

$$\Omega_b = 0.91107$$

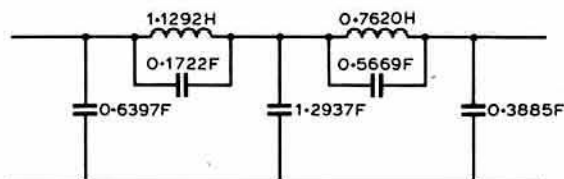


Fig 14. Component values for C 05 05 $\theta = 43^\circ$

For the right-hand branch,

$$L = 9.2632H$$

$$C = 6.8915F$$

$$\beta = 1.13324$$

$$L_a = 0.06802H$$

$$L_b = 0.07709H$$

$$C_a = 12.9727F$$

$$C_b = 14.7012F$$

$$\Omega_a = 1.06454$$

$$\Omega_b = 0.93938$$

These transformations are shown in Fig 17.

At this point it is worthwhile checking the calculations.

First,

$$Q_{bp} (\Omega_a - \Omega_b) = \omega_a \text{ or } \omega_b.$$

For the RHS, $Q_{bp} = 12.1564$, $\Omega_a = 1.06454$, $\Omega_b = 0.93938$

$$12.1564 (1.06454 - 0.93938) = 1.5215$$

$$= \omega_a (1.5213)$$

For the LHS, $12.1564 (1.09762 - 0.91107) = 2.26778$

$$= \omega_b (2.2672)$$

Now the filter is scaled to the centre frequency of 3.6469MHz and, assuming a system impedance of 50 Ω , the final filter design is shown in Fig 18.

Choice of components

Passive filters are suitable for frequencies from 100kHz upwards; below this frequency inductor values become large, entailing bulky and often lossy coils. Active IF filters remove the need for inductors completely by using operational amplifiers.

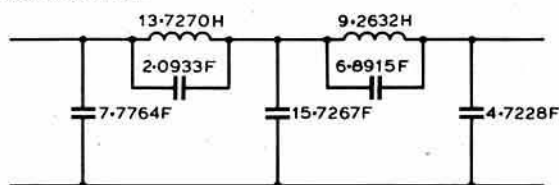


Fig 15. C 05 05 $\theta = 43^\circ$ component values multiplied by Q_{bp}

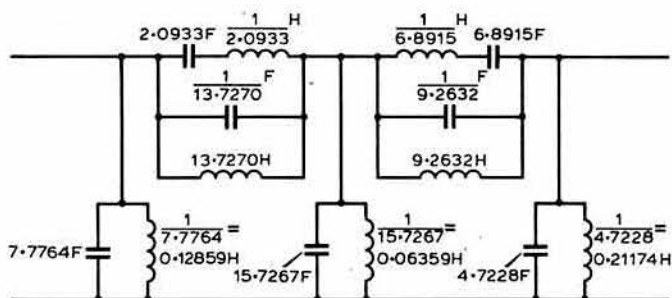


Fig 16. Normalized bandpass filter

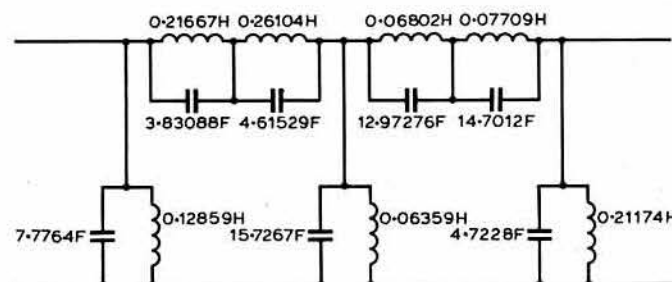


Fig 17. Normalized bandpass filter with transformed upper branches

Silver mica, polystyrene or low-K ceramic capacitors are most suitable with their one to two per cent accuracy ensuring good repeatability of the design. The author has found toroidal inductors to be most useful, providing high-Q and self-screening properties. For single layer and pile-wound coils, the correct ferrite core will enable a high-Q to be obtained and assist adjustments.

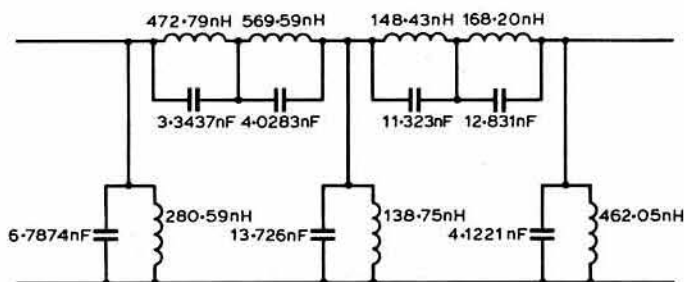


Fig 18. Final scaled circuit

Optimum results will be obtained with highly accurate values of components. It is wise to check the values of inductors before they are placed in the circuit, since turn spacing and lead lengths can cause errors. This can be done using a gdo and a frequency counter. Accurate capacitor values can be made by series and parallel combinations, remembering that the accuracy cannot be better than the least accurate component.

Bibliography

- Electronic Filter Design Handbook*, A. B. Williams. Mc-Graw Hill, 1981.
- Filter Design And Evaluation*, G. E. Hansell. Reinhold Book Corp, 1969.
- Handbook of Filter Design*, R. Saal. AEG-Telefunken, 1979.
- Handbook Of Filter Synthesis*, A. I. Zverev. John Wiley & Sons, 1967.
- Simplified Modern Filter Design*, P. Geffe. John F. Rider, 1963.

RSGB AMATEUR RADIO CALL BOOK (1983 edn)

The much-expanded 1983 edition of this invaluable directory of UK and Republic of Ireland amateur radio stations incorporates over 10,700 new call signs and amendments notified to the RSGB by the Home Office and the Irish Radio Transmitters Society between August 1981 and July 1982. It also includes lists of RSGB affiliated societies and groups, plus RSGB repeaters and special call signs.

228 pages

273 by 204mm